EF04 THE ELEMENTARY CHARGE & MOTION IN ELECTRIC FIELDS

SPH4U

CH 7 – KEY IDEAS

- define and describe concepts and units related to electric and gravitational fields
- state Coulomb's law and Newton's law of universal gravitation, and analyze, compare, and apply them in specific contexts
- compare the properties of electric and gravitational fields by describing and illustrating the source and direction of the field in each case
- apply quantitatively the concept of electric potential energy and compare it to gravitational potential energy
- analyze quantitatively, and with diagrams, electric fields and electric forces in a variety of situations
- describe and explain the electric field inside and on the surface of a charged conductor and how the properties of electric fields can be used to control the electric field around a conductor
- perform experiments or simulations involving charged objects
- explain how the concept of a field developed into a general scientific model, and describe how it affected scientific thinking

EQUATIONS

- Elementary Charge (charge of an electron)

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$

- Charge in terms of elementary charge

$$Q = Ne$$

- Work on a charged particle

$$W = \Delta V q = -\Delta E_E = \Delta E_K$$

MILLIKAN OIL DROP

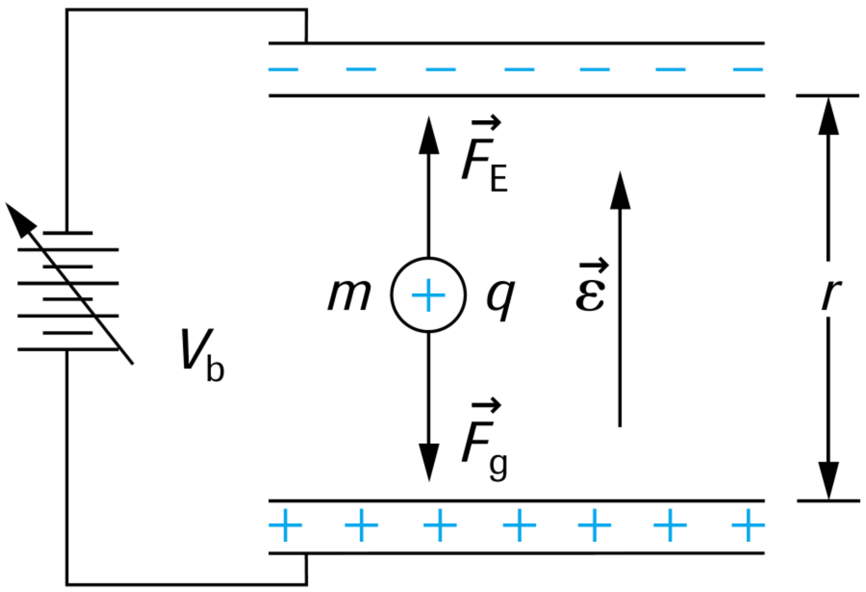
- For a positively charged drop of mass m and charge q, the force upwards from the positively charged lower plate is

$$\vec{F}_E = q\vec{\varepsilon}$$

- If a drop is suspended between the plates, its net force is zero, giving

$$F_E = F_g$$

$$q\varepsilon = mg$$



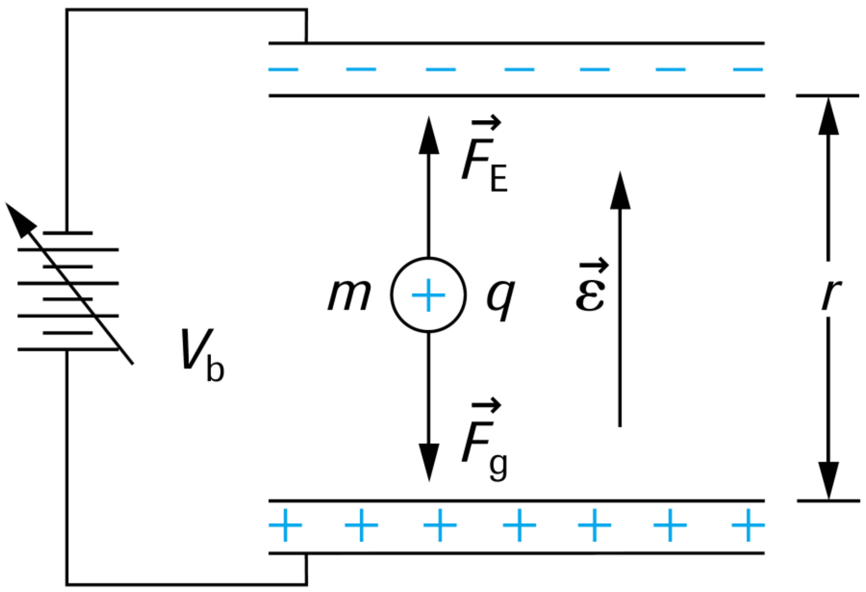
MILLIKAN OIL DROP – CONT.

- Recall: $\varepsilon = \frac{\Delta V}{r}$
- If we call the balancing potential difference ΔV_b

$$q = \frac{mg}{\varepsilon}$$

$$q = \frac{mgr}{\Delta V_b}$$

- Millikan Oil Drop Interactive Activity



MILLIKAN OIL DROP – CONT.

- Millikan measured
  - the required potential difference between the plates
  - the terminal velocity to determine the mass of the drop
  - the distance between the plates
- Using this information, he could calculate the charge of the oil drop, which is an integer multiple of the elementary charge
- He determined the charge of an electron (elementary charge) is

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$

- A charged object with a difference of N electrons has charge

$$\boldsymbol{Q = Ne}$$

- Millikan Oil Drop Video

PROBLEM 1

Calculate the charge on a small sphere with an excess of 5.0×10^{14} electrons.

PROBLEM 1 – SOLUTIONS

$N = 5.0 \times 10^{14}$

$q = ?$

$$q = Ne$$
$$= (5.0 \times 10^{14})(1.6 \times 10^{-19}\ \text{C})$$
$$q = 8.0 \times 10^{-5}\ \text{C}$$

The charge on the sphere is -8.0×10^{-5} C (negative because of the excess of electrons).

PROBLEM 2

In a Millikan-type experiment, two horizontal plates are 2.5 cm apart. A latex sphere, of mass 1.5×10^{-15} kg, remains stationary when the potential difference between the plates is 460 V with the upper plate positive.

(a) Is the sphere charged negatively or positively?

(b) Calculate the magnitude of the charge on the latex sphere.

(c) How many excess or deficit electrons does the sphere have?

PROBLEM 2 – SOLUTIONS

$r = 2.5\ \text{cm}$ $\quad$ $q = ?$

$m = 1.5 \times 10^{-15}\ \text{kg}$ $\quad$ $N = ?$

$\Delta V = 460\ \text{V}$

(a) The electric force must be up, to balance the downward force of gravity. Since the upper plate is positive, the latex sphere must be charged negatively to be attracted to the upper plate and repelled by the lower plate. The electric field is downward, giving an upward force on a negative charge.

PROBLEM 2 – SOLUTIONS CONT.

(b) When the sphere is balanced,

$$F_E = F_g$$
$$q\varepsilon = mg$$

But $\varepsilon = \dfrac{\Delta V}{r}$. Therefore,

$$\frac{q\Delta V}{r} = mg$$
$$q = \frac{mgr}{\Delta V}$$
$$= \frac{(1.5 \times 10^{-15}\ \text{kg})(9.8\ \text{m/s}^2)(2.5 \times 10^{-2}\ \text{m})}{460\ \text{V}}$$
$$q = 8.0 \times 10^{-19}\ \text{C}$$

The magnitude of the charge is 8.0×10^{-19} C.

PROBLEM 2 – SOLUTIONS CONT.

(c)

$$N = \frac{q}{e}$$

$$= \frac{8.0 \times 10^{-19}\ \text{C}}{1.6 \times 10^{-19}\ \text{C}}$$

$$N = 5$$

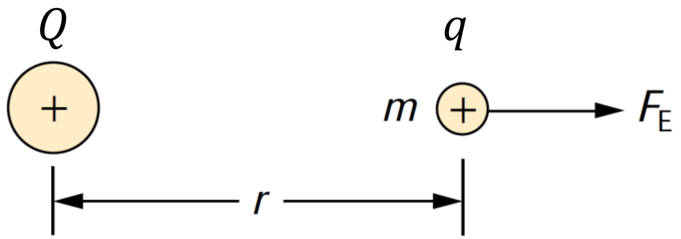
The sphere has 5 excess electrons (since the charge is negative).

NOTES ON THE FUNDAMENTAL CHARGE

- Charge of a proton is equal in magnitude to the charge of an electron
  - Protons can be broken down into quarks (more on that later) which divide the charge
  - Not “fundamental” as quarks do not exist on their own
- Charges of larger particles are always integer multiples of the fundamental charge
- Unlike energy, charge cannot be transformed and is always conserved in nuclear and chemical reactions

ACCELERATION OF CHARGED PARTICLES

- Recall: $F_E = \frac{kQq}{r^2}$
- From Newton's 2nd Law



$$a = \frac{F_E}{m}$$

  - a – instantaneous acceleration of the particle [m/s^2]
  - F_E – electric force of charge Q on charge q [N]
  - m – mass of charge q [kg]
- However, since force changes with distance, so too does acceleration, making it difficult to calculate

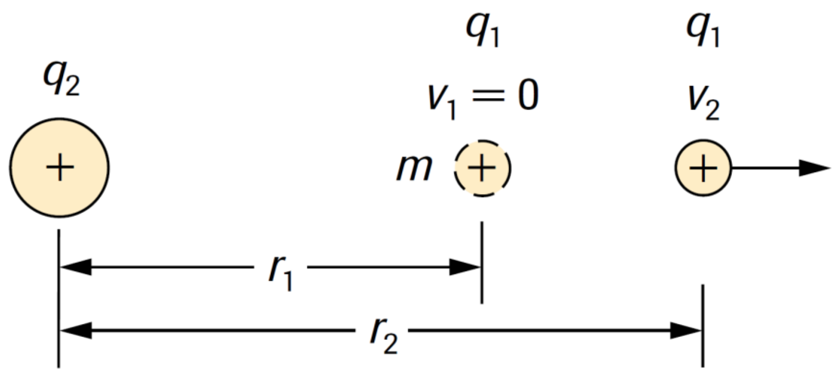
CHANGE IN KINETIC ENERGY OF CHARGED PARTICLES

- Using conservation of energy, we can determine how the particle moves
- The total energy of the particle is contant
- Since the masses are very small, we can neglect gravitational potential energy

$$E = E'$$
$$E_K + E_E = E'_K + E'_E$$
$$E_E - E'_E = E'_K - E_K$$
$$-\Delta E_E = \Delta E_K$$

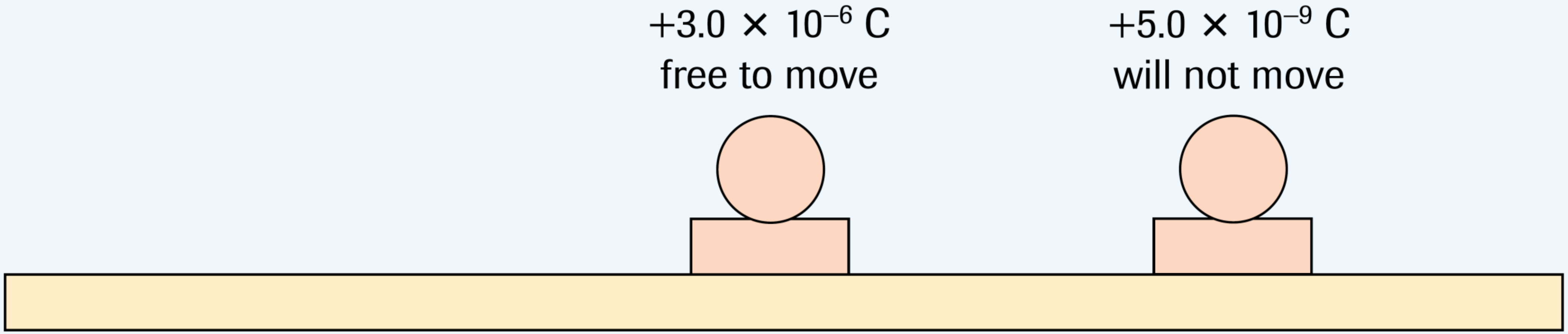
- Since the particle starts at rest, this means

$$-\Delta E_E = E'_K$$



PROBLEM 3

Figure 3 shows two small conducting spheres placed on top of insulating pucks. One puck is anchored to the surface, while the other is allowed to move freely on an air table. The mass of the sphere and puck together is 0.15 kg, and the charge on each sphere is $+3.0 \times 10^{-6}$ C and $+5.0 \times 10^{-9}$ C. The two spheres are initially 0.25 m apart. How fast will the sphere be moving when they are 0.65 m apart?



PROBLEM 3 – SOLUTIONS

$m = 0.15$ kg $\qquad$ $r = 0.25$ m

$q_1 = +3.0 \times 10^{-6}$ C $\qquad$ $r' = 0.65$ m

$q_2 = +5.0 \times 10^{-9}$ C $\qquad$ $v' = ?$

To find the speed, the kinetic energy at $r' = 0.65$ m is required. Since the initial kinetic energy is 0, the final kinetic energy is equal to the change in kinetic energy. To determine the change in kinetic energy, we first find the change in electric potential energy:

$$
\begin{aligned}
E_K' &= \Delta E_K \\
&= -\Delta E_E \\
&= -\left(\frac{kq_1q_2}{r'} - \frac{kq_1q_2}{r}\right) \\
&= -\frac{(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{0.65\ \text{m}} + \frac{(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{0.25\ \text{m}} \\
E_K' &= 3.3 \times 10^{-4}\ \text{J}
\end{aligned}
$$

PROBLEM 3 – SOLUTIONS CONT.

We can now find the speed:

$$v' = \sqrt{\frac{2E_K'}{m}}$$

$$= \sqrt{\frac{2(0.33\ \text{J})}{0.15\ \text{kg}}}$$

$$v' = 0.07\ \text{m/s}$$

The sphere will be moving with a speed of 0.07 m/s.

MOTION IN UNIFORM ELECTRIC FIELDS

- In a uniform electric field (between parallel plates)

$$\vec{F}_E = q\vec{\varepsilon} = \text{constant}$$

- From Newton's 2nd Law

$$\vec{a} = \frac{\vec{F}_E}{m} = \text{constant}$$

- This tells us a particle will move with constant acceleration in a uniform field

WORK DONE BY A CONSTANT FORCE

- Between two parallel plates, electric field is in the same direction as the motion
- The work done on a charge q from one plate to the other

$$W = \vec{F}_E \vec{r}$$

$$W = \varepsilon q r$$

$$W = \frac{\Delta V}{r} q r$$

$$\boldsymbol{W = \Delta V q = -\Delta E_E = \Delta E_K}$$

PROBLEM 4

The cathode in a typical cathode-ray tube (**Figure 4**), found in a computer terminal or an oscilloscope, is heated, which makes electrons leave the cathode. They are then attracted toward the positively charged anode. The first anode has only a small potential rise while the second is at a large potential with respect to the cathode. If the potential difference between the cathode and the second anode is 2.0×10^4 V, find the final speed of the electron.

PROBLEM 4 – SOLUTIONS

$\Delta V = 2.0 \times 10^4\ \text{V}$ $\quad q = 1.6 \times 10^{-19}\ \text{C}$

$m = 9.1 \times 10^{-31}\ \text{kg}$ $\quad v = ?$

For the free electron,

$$-\Delta E_E = \Delta E_K$$

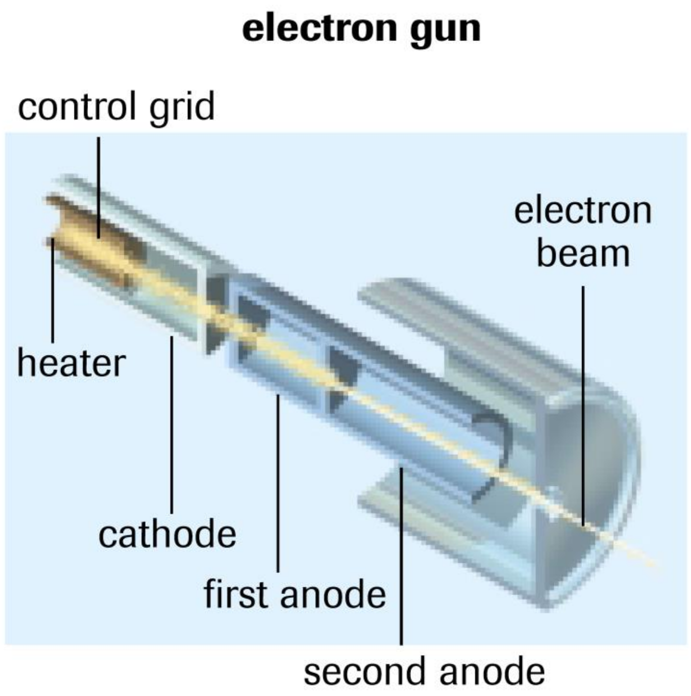
$$q\Delta V = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

$$= \sqrt{\frac{2(1.6 \times 10^{-19}\ \text{C})(2.0 \times 10^4\ \text{V})}{9.1 \times 10^{-31}\ \text{kg}}}$$

$$v = 8.4 \times 10^7\ \text{m/s}$$

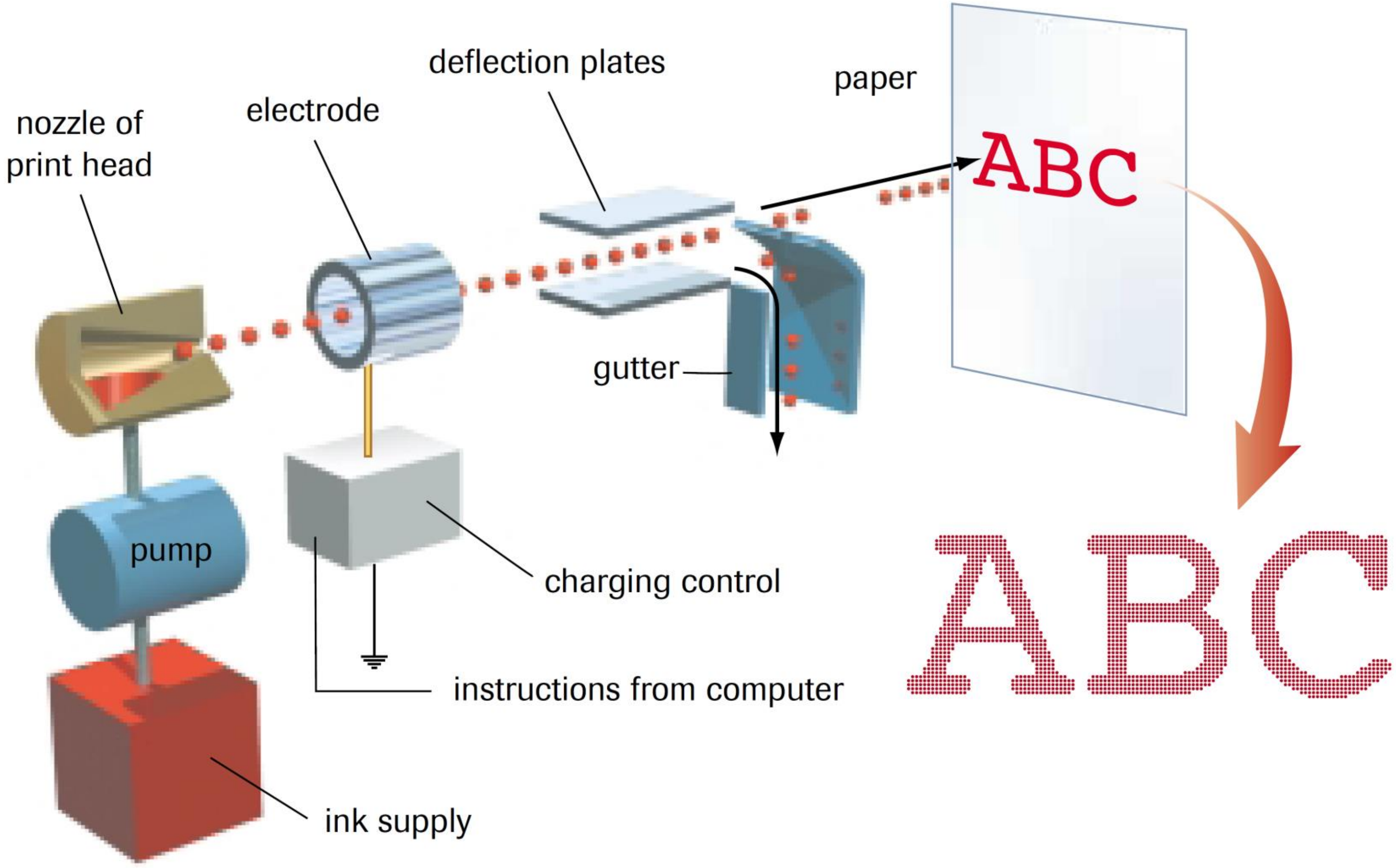
The final speed of the electron is 8.4×10^7 m/s.



INKJET PRINTERS

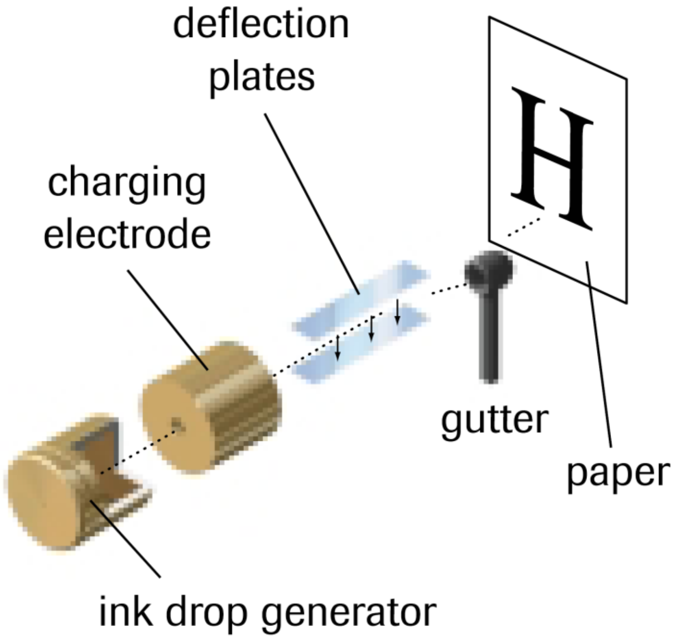
- Two types of inkjet printers use parallel plates to control the ink pattern printed onto paper
- One type controls when to charge droplets of ink to deflect them away from the paper with parallel plates
- All neutral droplets are left to make an image on the paper, while charged droplets are diverted to a gutter
- The parallel plates in this model are left constant and the electrode is variable

INKJET PRINTERS – CONT.



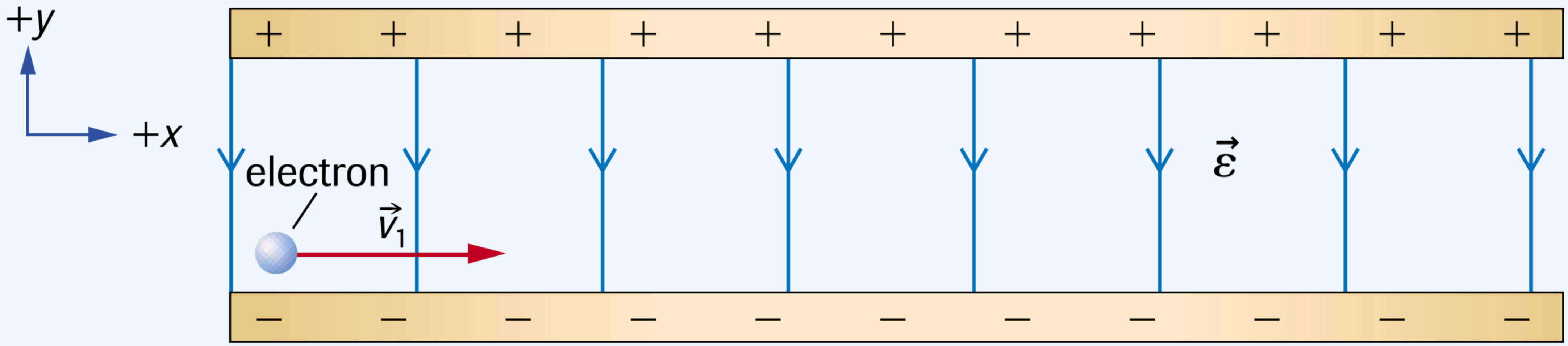
INKJET PRINTERS – CONT.

- Another type charges all the ink droplets
- The parallel plates control the direction of the charged ink droplets
- Charges on the plates will direct the droplets into the gutter when they are not part of the image
- In this model, the electrode is creating a constant charge and the parallel plates are variable



PROBLEM 5

An electron is fired horizontally at 2.5×10^6 m/s between two horizontal parallel plates 7.5 cm long, as shown in **Figure 7**. The magnitude of the electric field is 130 N/C. The plate separation is great enough to allow the electron to escape. Edge effects and gravitation are negligible. Find the velocity of the electron as it escapes from between the plates.



PROBLEM 5 – SOLUTIONS

$q = 1.6 \times 10^{-19}$ C $\quad\quad \varepsilon = 130$ N/C

$\vec{v}_1 = 2.5 \times 10^6$ m/s [horizontally] $\quad\quad \vec{v}_2 = ?$

$l = 7.5$ cm

Note that $\vec{v}_2$ has two components, v_{2x} and v_{2y}.

The magnitude of the electric field is constant, and the field is always straight down; therefore, the electric force on the electron is constant, meaning its acceleration is constant and vertical. We can break the problem up into a horizontal part, which involves just uniform motion, and a vertical part, which involves constant acceleration. There is no need to use energy here (although you could). We will use forces and kinematics instead.

PROBLEM 5 – SOLUTIONS CONT.

The magnitude of the net force on the electron is

$$\begin{aligned} F_{\text{net}} &= F_{\text{E}} \\ &= q\varepsilon \\ F_{\text{net}} &= e\varepsilon \end{aligned}$$

Therefore, the magnitude of the acceleration of the electron is given by

$$\begin{aligned} a_y &= \frac{e\varepsilon}{m_{\text{e}}} \\ &= \frac{(-1.6 \times 10^{-19}\ \text{C})(-130\ \text{N/C})}{9.1 \times 10^{-31}\ \text{kg}} \\ a_y &= 2.3 \times 10^{13}\ \text{m/s}^2 \\ \vec{a} &= 2.3 \times 10^{13}\ \text{m/s}^2\ [\text{up}] \end{aligned}$$

This acceleration is upward because the electron is repelled by the lower plate and attracted to the upper plate, as indicated by the direction of the electric field.

PROBLEM 5 – SOLUTIONS CONT.

The initial velocity in the vertical direction is zero. To find the final vertical velocity, it suffices to find the time spent on the vertical movement, which equals the time spent passing through the plates. That time, in turn, is given to us by the horizontal velocity and the width of the plates. (Remember that the electron moves with uniform motion in the horizontal direction.)

$$\Delta t = \frac{\Delta l}{v_x}$$

$$= \frac{7.5 \times 10^{-2}\ \text{m}}{2.5 \times 10^{6}\ \text{m/s}}$$

$$\Delta t = 3.0 \times 10^{-8}\ \text{s}$$

The final vertical component of the velocity is

$$v_{2y} = v_{1y} + a_y \Delta t$$

$$= 0 + (2.3 \times 10^{13}\ \text{m/s}^2)(3.0 \times 10^{-8}\ \text{s})$$

$$v_{2y} = 6.9 \times 10^{5}\ \text{m/s [up]}$$

PROBLEM 5 – SOLUTIONS CONT.

Adding these two components head-to-tail and using the Pythagorean theorem, as in **Figure 8**, gives

$$v_2 = \sqrt{(6.9 \times 10^5 \text{ m/s}) + (2.5 \times 10^6 \text{ m/s})}$$
$$v_2 = 2.6 \times 10^6 \text{ m/s}$$

We determine the angle upward from the horizontal:

$$\theta = \tan^{-1}\left(\frac{6.9 \times 10^5 \text{ m/s}}{2.5 \times 10^6 \text{ m/s}}\right)$$
$$\theta = 15°$$

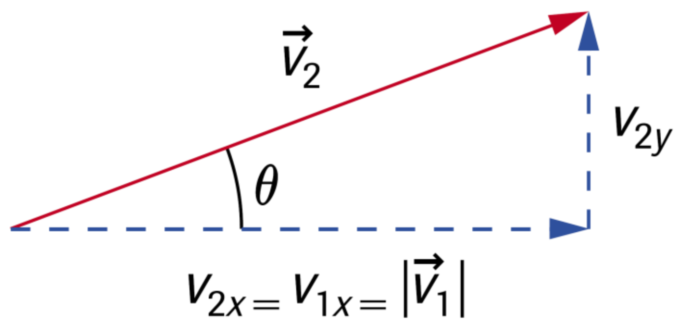


Figure 8

The final velocity is 2.6×10^6 m/s [right 15° up from the horizontal].

SUMMARY – DETERMINING THE ELEMENTARY CHARGE

- There exists a smallest unit of electric charge, called the elementary charge, *e*, of which other units are simple multiples

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$

SUMMARY – THE MOTION OF CHARGED PARTICLES IN ELECTRIC FIELDS

- A charged particle in a uniform electric field moves with uniform acceleration.
- From conservation principles, any changes to a particle's kinetic energy result from corresponding changes to its electric potential energy (when moving in any electric field and ignoring any gravitational effects).

PRACTICE

Readings

- Section 7.5 (pg 360)
- Section 7.6 (pg 365)

Questions

- pg 364 #2-5
- pg 371 #1-5